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STEADY-STATE HEAT CONDUCTION IN COMPOSITE SYSTEMS WITH BOUNDARY CONDITIONS OF THE FOURTH KIND

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A method is proposed for solving steady-state heat-conduction problems in a system of contacting regions, and examples are presented to illustrate its effectiveness.

As high-temperature thermal physics develops, problems of determining temperature distributions in systems of contacting bodies become more and more important [1]. We present a method for solving steady-state heat-conduction problems with matching boundary conditions based on the application of well-developed methods of solving elliptic differential equations in regions with piecewise homogeneous media [2, 3] and mathematical optimization methods (Hooke, Rosenbrock) [4].

The method involves the following steps.

1. A mathematical statement of the problem of determining the temperature distribution in a system of N contacting bodies is formulated.

2. Boundary conditions of the fourth kind on interfaces S_{ij} between the i-th and j-th regions of the original problem are replaced by boundary conditions of the second kind

$$\begin{gathered}\lambda_i \left.\frac{\partial U_i}{\partial n}\right|_{S_{ij}} = q_i(S_{ij}),\\ \lambda_j \left.\frac{\partial U_j}{\partial n}\right|_{S_{ij}} = q_j(S_{ij}),\\ q_i(S_{ij}) = -q_j(S_{ij}),\end{gathered} \tag{1}$$

where $q_i(S_{ij})$ and $q_j(S_{ij})$ are unknown heat flux distribution functions on the boundary S_{ij} between the i-th and j-th regions. In this way the original boundary-value problem for determining the temperature distribution in the system is separated into N independent problems.

3. It is assumed that the functions $q_i(S_{ij})$ can be expressed by polynomials or step function representations by using one of the known methods of constructing a solution in each region. The solutions obtained in this way are parametrically dependent on the coefficients Q_{ik} which appear in the functions $q_i(S_{ij})$ and also on the constants C_i for an internal Neumann problem.

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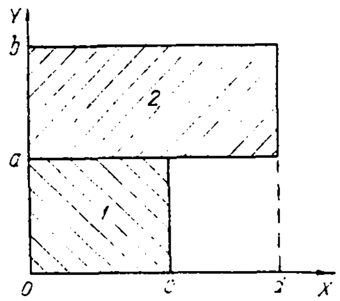

Fig. 1. System of two contacting bodies.

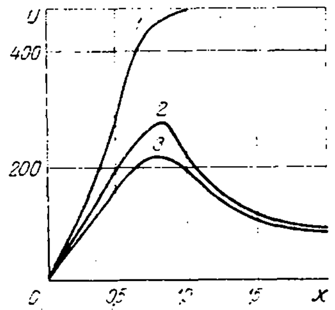

Fig. 2. Temperature distribution in system: 1) y = 0; 2) y = b; 3) y = c. U is in °K and x in m.

4. A target function is constructed and minimized by

$$F = \frac{1}{L}\sum_{S_{ij}}\frac{1}{S_{ij}}\int_{S_{ij}}|U_i(S)-U_j(S)|\,dS \tag{2}$$

choosing a set of Q^*_{ik}, C^*_i by using one of the methods for finding a local extremum. The magnitude of the minimum of the target function is the mean absolute error of matching the temperature distributions, and characterizes the accuracy of the solution of the original problem.

We illustrate the proposed method for solving steady-state heat-conduction problems in contacting regions by the following example:

It is required to solve Laplace's equation in the system of two bodies shown in Fig. 1:

$$\nabla^2 U_i = 0,\ i = 1,\ 2 \tag{3}$$

and to satisfy the boundary conditions

$$U_1|_{x=0,\ y\in(a,b)} = 0, \tag{4}$$

$$\left.\frac{\partial U_1}{\partial x}\right|_{x=d,\ y\in(a,\ b)} = 0, \tag{5}$$

$$\left.\frac{\partial U_1}{\partial y}\right|_{y=a,\ x\in(c,\ d)} = 0, \tag{6}$$

$$\left.\left(\lambda_1\frac{\partial U_1}{\partial y}+\alpha U_1\right)\right|_{y=b,\ x\in(0,\ d)} = 0, \tag{7}$$

$$\left.\left(\lambda_1\frac{\partial U_1}{\partial y}-\lambda_2\frac{\partial U_2}{\partial y}\right)\right|_{y=a,\ x\in(0,\ c)} = 0, \tag{8}$$

$$(U_1-U_2)|_{y=a,\ x\in(0,\ c)} = 0, \tag{9}$$

$$U_2|_{x=0,\ y\in(0,\ a)} = 0, \tag{10}$$

$$\left.\frac{\partial U_2}{\partial x}\right|_{x=c,\ y\in(0,\ a)} = 0, \tag{11}$$

$$-\lambda_2\left.\frac{\partial U_2}{\partial y}\right|_{y=0,\ x\in(0,\ c)} = f(x) = \begin{cases}0, & x\in(0,\ c/2),\\ f_0, & x\in(c/2,\ c).\end{cases} \tag{12}$$

In accord with Eq. (1) we replace the boundary conditions of the fourth kind in the formulation of problem (3)-(12) by boundary conditions of the second kind

$$\lambda_1 \left.\frac{\partial U_1}{\partial y}\right|_{y=a,\ x\in(0,\ c)} = q(x), \tag{13}$$

$$\lambda_2 \left.\frac{\partial U_2}{\partial y}\right|_{y=a,\ x\in(0,\ c)} = q(x). \tag{14}$$

We replace the function q(y) by the step function representation

$$q(x) = q_j\ x \in ((j-1)c/M,\ jc/M),\ j = 1, 2, \ldots, M. \tag{15}$$

Using the Fourier method [3] we construct a solution of the boundary-value problems in the individual regions which takes account of (13)-(15)

$$U_1(x,\ y) = \frac{-4}{\lambda_1 c} \sum_{k=0}^{\infty} \left\{ \frac{\sin\left(\frac{\mu_k a}{4}\right)}{\mu_k^2} \left(\operatorname{sh}(\mu_k y) - \frac{\operatorname{ch}(\mu_k y)}{\operatorname{th}(\mu_k b)} \right) \times \right.$$
$$\left. \times\ \lambda_0 \sin\left(\frac{3\mu_k a}{4}\right) + \frac{\sin\left(\frac{\mu_k a}{2M}\right)}{\mu_k^2} \frac{\operatorname{ch}(\mu_k y)}{\operatorname{sh}(\mu_k b)} \sum_{i=1}^{M} \left[q_i \sin\left(\frac{(2i-1)\mu_k a}{2M} \right) \right] \right\} \sin(\mu_k x), \tag{16}$$

where

$$\mu_k = \frac{\pi(2k+1)}{2a},$$

$$U_2(x,\ y) = \frac{-4}{\lambda_2 d} \sum_{k=0}^{\infty} \frac{\sin\left(\frac{\xi_k a}{2M}\right)}{\xi_k^2} \frac{\operatorname{ch}(\xi_k y) - \nu_k \operatorname{sh}(\xi_k y)}{\operatorname{sh}(\xi_k b) - \nu_k \operatorname{ch}(\xi_k q)} \sum_{i=1}^{M} b_i \sin\left(\frac{(2i-1)\xi_k a}{2M} \right) \sin(\xi_k x); \tag{17}$$

here

$$\xi_k = \frac{\pi(2k+1)}{2d}, \qquad \nu_k = \frac{\lambda_2 \xi_k \operatorname{th}(\xi_k c) + \alpha}{\lambda_2 \xi_k + \alpha \operatorname{th}(\xi_k c)}.$$

We seek a set of q^*_j to minimize the target function

$$F = \frac{1}{c} \int_0^c |U_1(x,\ a) - U_2(x,\ a)|\, dx$$

in the M-dimensional parallelepiped $[q^L_j,\ q^H_j]$, $1 \le j \le M$. To do this we use the Rosenbrock method, which is a certain development of the Gauss—Seidel method [4]. Substitution of the

TABLE 1. Temperature Distribution for c = d

$x \cdot 10^{-3}$, m	T_a, °K, $y=0$	T_{app}, °K $y=0$	T_a, °K, $y=a$	T_{app}, °K $y=a-0$	T_{app}, °K $y=a+0$	T_a, °K, $y=b$	T_{app}, °K $y=b$
1	53,23	52,52	49,44	49,41	49,42	45,51	45,82
2	106,72	105,37	99,60	96,55	101,07	91,21	91,81
3	168,84	168,30	151,10	148,68	152,25	137,02	137,71
4	235,32	234,20	204,05	200,64	205,70	182,26	182,75
5	347,72	347,46	256,84	257,97	258,23	225,41	225,49
6	453,67	454,05	305,00	307,43	303,76	264,23	264,02
7	507,13	507,64	343,98	343,66	344,14	296,43	296,18
8	543,55	544,08	371,96	373,12	371,40	320,31	320,07
9	562,79	563,30	388,68	388,70	388,69	334,92	334,70
δ, %	1,34			3,30		0,68	

Note: T_a) analytical solution; T_{app}) approximate solution given by Eqs. (16) and (17); δ) maximum discrepancy.

values found for the q^*_j into Eqs. (16) and (17) gives an approximate solution of problem (3)-(12).

Figure 2 shows the results of a numerical calculation of the temperature distribution in the system calculated on a BÉSM-6 computer for the following values of the parameters: $\lambda_1 = 200$ W/m•deg, $\lambda_2 = 100$ W/m•deg, $a = 2\cdot10^{-3}$ m, $b = 4\cdot10^{-3}$ m, $c = 10^{-2}$ m, $d = 2\cdot10^{-2}$ m, $\alpha = 10^4$ W/m^2•deg, $f_0 = 10^5$ W/m^2, M = 5. The lower limit of the variation of the variables was taken as $q^L{}_j = -f_0$, the upper limit $q^U{}_j = f_0$, and the initial approximation $q_j = 0$.

In order to compare the approximate solution found by the scheme described by steps 1-4 with the analytical solution, problem (3)-(12) was solved for d = c. The maximum discrepancy between the analytical solution and that obtained by using the proposed scheme does not exceed 3.3% (Table 1). The solution of this problem by the net-point method using the complex of codes of the KSI-BÉSM-6 program [5] is much inferior to the proposed method in the expenditure of machine time for the same accuracy of the solution.

NOTATION

U_i, U_j, temperature distribution functions in i-th and j-th regions; n, unit vector in direction of outward normal to boundary of region; λ_i, λ_j, thermal conductivities of i-th and j-th regions; L, number of matching boundaries; x, y, axes of Cartesian coordinate system; α, heat-transfer coefficient; f(x), heat flux distribution function; M, number of coefficients in step function representation of q(x).

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APPLICATION OF DIMENSIONAL ANALYSIS TO THE PROBLEM OF THE ACTION OF ULTRASOUND ON AIR IN A CAPILLARY TUBE

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By using dimensional analysis an expression is obtained for the air pressure change in a capillary tube channel caused by the effect of ultrasound. A comparison with experimental results is performed.

An analysis of the dimensionality of the quantities governing a physical phenomenon affords a possibility, in a number of cases, of obtaining characteristic relationships comparatively easily, which connect these quantities in a mathematical description of the phenomenon [1]. Dependences of the air pressure in channels of dead-end capillary tubes placed upright at a short distance from the concentrator on different parameters were experimentally obtained in [2]. It was shown that the main variables on which the P in the channel depends are the amplitude of the concentrator A displacement and the frequency of the ultrasonic oscillations f, the inner tube diameter d, and its wall thickness Δ, as well as the magnitude of the effective gap δ*, which equals the distance between the tube endface and the lower position of the oscillating concentrator. Moreover, characteristics of the medium in which the ultrasonic oscillations are propagated and the flow which causes the change in pressure

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